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ACTIVE CONTROL OF A CANTILEVER BEAM SUBJECT TO PARAMETRIC EXCITATION VIA NEGATIVE FEEDBACK VELOCITY

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ABSTRACT

The dynamical behavior of a cantilever beam subject to parametric excitation is investigated. The method of multiple scale perturbation technique is applied to obtain the solution up to the second order approximation. At first we studied the system behavior without control and the effect of parameters at this case. The active control of negative feedback with linear and cubic velocity is used to vibration control. It can be seen that the linear control is more effective than the cubic control. All resonance cases are studied numerically to obtain the worst case. Stability of the steady state solution for the selected resonance case is studied applying Rung-Kutta fourth method and frequency response equation via Matlab 7.0 and Maple 16.

KEYWORDS: Cantilever Beam, Active Control, Vibration Control, Resonance Cases & Parametric Excitation

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Original Article

1. INTRODUCTION

Over the last few years, numerous papers have been dedicated to the control of resonantly forced systems in various engineering fields. Comparing active control to passive control, different studies prove that the later is more sophisticated than the former in various points. As in passive vibration absorbers, a physical device is connected to the primary structure, on the contrary, in active absorbers, the device is replaced by a control system of sensors, actuators and filters.

Oueni et al. [1] studied a non-linear active vibration absorber coupled with the plant through user-defined cubic nonlinearities. Tondl and Nbergo [2] studied a dynamic absorber, which can move in the transverse or longitudinal direction, which attached to an externally excited simple pendulum. Nayfeh et al. [3] discussed dynamics of machining using quadratic and cubic stiffness of machine tools, which accounts for the regenerative effects.

Hamdi and Belhaq [4] investigated nontrivial solutions and bistability in the Duffing oscillator with a delayed displacement feedback via the perturbation method. Eissa and co workers [5-7], studied the vibration reduction of a ship pitch-roll motion described by non-linear spring pendulum system under multi external, parametric excitations or both of them can be reduced using a longitudinal absorber. Eissa et al. [8-9] studied the active control of an aircraft tail subjected to multiharmonic excitation or multi-parametric excitation forces. They added many active controllers for this system. The system has controlled by negative (linear, quadratic) velocity feedback. Best active control of the system has been achieved via negative velocity feedback.

Y. S. Hamed et al. [10-12] studied the USM model subject to external and tuned excitation forces. The model consists of multi-degree-of-freedom system consisting of the tool holder and three absorbers (tools) simulating ultrasonic machining process. The advantages of using multi-tools are to machine different materials and different shapes at the same time. This leads to time saving and higher machining efficiency. Warminski et al. [13] analyzed the influence of the controller parameters, and also the hard or soft geometrical plant nonlinearities, to the dynamical response of the system undergoing NSC. Moreover, the effectiveness of the selected control algorithms to vibration suppression of a strongly nonlinear beam structure was compared and discussed.

Lei et al. [14] applied an active control technique to coordinate a kind of two parametrically excited chaotic system. Oueini and Nayfeh [15] modelled the dynamics of the first mode of a cantilever beam with a second order nonlinear ordinary differential equation subjected to a principal parametric excitation, and a control law based on cubic velocity feedback is introduced. Pai and Schulz [16] studied the control of the first mode vibration of a stainless steel beam through negative velocity feedback to the dynamic system.

Sayed and Kamel [17-18] studied the effect of different controllers on the vibrating system and the saturation control of a linear absorber to reduce vibrations due to rotor blade flapping motion. The stability of the numerical solution is investigated using both phase plane methods and frequency response equations. J. Warminski [19-20] studied apart from the regular dynamics, interactions between self- parametric or external excitations may lead to chaotic or hyperchaotic dynamics.

Saeed et al [21] studied nonlinear time delay saturation-based controller for suppression of nonlinear beam vibrations. Gao and Chen [22] studied Active vibration control for a Bilinear system with nonlinear velocity time-delayed feedback. Yingli et al [23] studied Dynamic effects of delayed feedback control on nonlinear vibration floating raft systems. Gao, and Chen [24] studied Nonlinear analysis, design and vibration isolation for a bilinear system with time-delayed cubic velocity feedback. Amer and Ahmed [25] studied Vibration control of a nonlinear dynamical system with time varying stiffness subjected to multi external forces

The objective of this work is to study dynamical behavior of a cantilever beam subject to parametric excitation under state feedback active control. The method of multiple scale perturbation technique is applied to obtain the solution up to the second order approximation.

The equation of the dynamical behavior of a cantilever beam subject to parametric excitation is given by:

$$\ddot{X}+\varepsilon\mu\dot{X}+X+\varepsilon\alpha X^{3}+\varepsilon\beta\dot{X}X^{2}+\varepsilon\gamma\ddot{X}X^{2}=T+\varepsilon X f\cos(\phi t) \tag{1}$$

where a dot denotes differentiation with respect to time, μ the damping coefficient, α the curvature nonlinearity coefficients, β and γ are the inertia nonlinearity coefficients, f is the amplitude of the parametric excitation, ϕ is the corresponding frequency, $T=-\varepsilon v\dot{X}^{r}$, $r=1,3.$ is the control v is the feedback gain.

2. MATHEMATICAL TECHNIQUES

The multiple scale perturbation method is conducted to obtain the approximate solution of Eq. (1). Assuming the solution in the form:

$$X(t;\varepsilon)=x_0(T_0,T_1)+\varepsilon\, x_1(T_0,T_1)+\varepsilon^2 x_2+O(\varepsilon^3) \tag{2}$$

where $T_0=t$ and $T_n=\varepsilon^n t$, (n=1,2) are the fast and slow time scales, respectively. In terms of T_0, T_1 *and* T_2, the times derivatives transform are:

$$\frac{d}{dt}=D_0+\varepsilon D_1\ ,\frac{d^2}{dt^2}=D_0^2+2\varepsilon D_0 D_1+\varepsilon^2(D_1^2+2D_0 D_2) \tag{3}$$

Where $D_n=\dfrac{\partial}{\partial T_n}$, (n=0,1,3).

Substituting Eqs. (2) and (3) into Eq. (1) and equating the coefficients of the same power of ϵ in both sides, we obtain:

$$O(\varepsilon^0):(D_0^2+1)x_0=0 \tag{4}$$

$$O(\varepsilon):(D_0^2+1)x_1=-2D_0D_1x_0-\mu D_0x_0-\alpha x_0^3-\beta x_0^2D_0x_0-\gamma x_0^2D_0^2x_0-vD_0x_0+f\,x_0\cos(\phi t) \tag{5}$$

$O(\varepsilon^2)$:

$$\begin{aligned}(D_0^2+1)x_2=&-2D_0D_1x_1-D_1^2x_0-2D_0D_2x_0-\mu D_0x_1-\mu D_1x_0-3\alpha x_0^2x_1-\beta x_0^2D_0x_1-\beta x_0^2D_1x_0\\&-2\beta x_0x_1D_0x_0-\gamma x_0^2D_0^2x_1-2\gamma x_0^2D_0D_1x_0-2\gamma x_0x_1D_0^2x_0-2vD_1x_0+f\,x_1\cos(\phi t)\end{aligned} \tag{6}$$

The general solution of Eq. (4) is

$$x_0(T_0,T_1)=A(T_1)e^{iT_0}+\overline{A}(T_1)e^{-iT_0} \tag{7}$$

where A is a complex function in T_1, which can be determined from eliminating the secular terms at the next approximation. Substituting Eq. (7) into Eq. (5), then we have that:

$$\begin{aligned}(D_0^2+1)x_1=&(-2iD_1A-i\mu A-3\alpha A^2\overline{A}-i\beta A^2\overline{A}+3\gamma A^2\overline{A}-ivA)e^{iT_0}+\\&(-\alpha A^3-i\beta A^3+\gamma A^3)e^{3iT_0}+\frac{f}{2}(Ae^{i(1+\phi)T_0}+\overline{A}e^{i(-1+\phi)T_0})+cc\end{aligned} \tag{8}$$

where cc denotes the complex conjugate, after elimination the secular terms then x_1 can be written as:

$$x_1(T_0,T_1)=E_1e^{3iT_0}+E_2e^{i(1+\phi)T_0}+E_3e^{i(-1+\phi)T_0}+cc \tag{9}$$

Substituting Eq. (7) and (9) into Eq. (6), then we have that:

$$
\begin{aligned}
(D_0^2+1)x_2 = {} & [-D_1^2A-2iD_2A-\mu D_1A-3\alpha E_1\bar{A}^2A^3-i\beta E_1\bar{A}^2A^3-\beta(D_1A)A^2-2\beta(D_1A)A\bar{A}+11\gamma E_1A^3\bar{A} \\
& -4i\gamma(D_1A)A\bar{A}+2i\gamma(D_1A)A^2-v(D_1A)-\frac{1}{2}E_2fA]e^{iT_0}+[-6iE_1A^3-3iE_1\mu A^3-6E_1\alpha A^4\bar{A} \\
& -6iE_1A^4\bar{A}+22E_1\gamma A^4\bar{A}-2i\gamma(D_1A)A^2-\beta(D_1A)A^2-3iE_1vA^3]e^{3iT_0}[-3E_1\alpha A^5-5iE_1\beta A^5 \\
& +11E_1\gamma A^5]e^{5iT_0}+[-2iE_2(1+\phi)A-iE_2\mu(1+\phi)A+6E_2\alpha A^2\bar{A}+4iE_1\beta(1+\phi)A^2\bar{A} \\
& -2E_2\gamma(1+\phi)^2A^2\bar{A}-2E_2\gamma A^2\bar{A}+iE_2v(1+\phi)]e^{i(1+\phi)T_0}+[3E_2\alpha\bar{A}^2A-2iE_2\beta(1+\phi)\bar{A}^2A \\
& -2iE_2\beta\bar{A}^2A-E_2\gamma(1+\phi)^2\bar{A}^2A-2E_2\gamma\bar{A}^2A]e^{i(-1+\phi)T_0}+\frac{1}{2}E_1fA^3e^{i(3-\phi)T_0}+[3E_2\alpha A^3 \\
& +iE_2\beta(1+\phi)A^3+2iE_2A^3-E_2\gamma(1+\phi)^2A^3-2E_2\gamma A^3+\frac{1}{2}E_1fA^3]e^{i(3+\phi)T_0} \\
& -\frac{1}{2}E_2fAe^{i(1+2\phi)T_0}
\end{aligned} \tag{10}
$$

Elimination the secular terms of Eq (10), then the solution of x_2 can be written as:

$$
\begin{aligned}
x_2(T_0,T_1) = {} & E_4e^{3iT_0}+E_5e^{5iT_0}+E_6e^{i(1+\phi)T_0}+E_7e^{i(-1+\phi)T_0}+E_8e^{i(3-\phi)T_0}+E_9e^{i(3+\phi)T_0} \\
& +E_{10}e^{i(1+2\phi)T_0}+cc
\end{aligned} \tag{11}
$$

where $E_j,(j=1,...,10)$ are complex functions of T_1.

3. STABILITY ANALYASIS

We study the different resonance numerically to see the worst resonance, one of the worst cases has been chosen to study the system stability. The selected resonance case $\phi = 2$. In this case we introduce the detuning parameter σ according to

$$\phi = 2 + \varepsilon\sigma \tag{12}$$

Substituting Eq. (12) into Eq. (8), then eliminating the secular and small divisor terms from x_1, we get the following

$$-2iD_1A - i\mu A - 3\alpha A^2\bar{A} - i\beta A^2\bar{A} + 3\gamma A^2\bar{A} - ivA + \frac{f}{2}\bar{A}e^{i\sigma T_1} = 0 \tag{13}$$

To analyze the solution of equation (13) it is convenient to express A in the polar form:

$$A(T_1) = \frac{1}{2}a(T_1)e^{i\theta(T_1)} \tag{14}$$

where a and θ and are unknown real-valued functions. Inserting Eq. (14) into Eq. (13) and separating real and imaginary parts, we have

$$a\theta' = \frac{3}{8}(\alpha-\gamma)a^3 - \frac{f}{4}a\cos\varphi \tag{15}$$

$$a' = -\frac{\mu}{2}a - \frac{\beta}{8}a^3 - \frac{v}{2}\mathrm{a} + \frac{f}{4}\mathrm{a}\sin\varphi \tag{16}$$

where $\varphi = \sigma T_1 - 2\theta$

For steady state solutions $a' = \varphi' = 0$, $\theta' = \frac{1}{2}\sigma$ obtained

$$f\cos\varphi = \frac{3}{2}(\alpha - \gamma)a^2 - 2\sigma \tag{17}$$

$$f\sin\varphi = 2\mu + \frac{a^2}{2}\beta + 2v \tag{18}$$

From equations (17-18) the frequency response equation is given by:

$$(\frac{3}{2}(\alpha-\gamma)a^2 - 2\sigma)^2 + (2\mu + \frac{\beta}{2}a^2 + 2v)^2 - f^2 = 0 \tag{19}$$

3.1 Linear Solutions

Now, to study the stability of the linear solution of the obtained fixed let us consider A in the form:

$$A(T_1) = \frac{1}{2}(p - iq)e^{i\delta T_1} \tag{20}$$

where p and q are real values functions in T_1 and consider $2\delta = \sigma$.

Substituting from Eq. (20) into the linear parts of Eq. (13) and separating real and imaginary parts, the following system of equations are obtained:

$$p' + \frac{1}{2}(\mu + \mathrm{v})p + (\delta - \frac{1}{4}f)q = 0 \tag{21}$$

$$q' + (-\delta - \frac{f}{4})p + \frac{1}{2}(\mu + v)q = 0 \tag{22}$$

$$\begin{vmatrix} -\frac{1}{2}(\mu+\mathrm{v}) - \lambda & -(\delta - \frac{1}{4}f) \\ (\delta + \frac{1}{4}f) & -\frac{1}{2}(\mu+\mathrm{v}) - \lambda \end{vmatrix} = 0 \tag{23}$$

where:

$$\lambda_{1,2} = -(\mu + v) \pm \frac{1}{4}\sqrt{64\delta^2 - f^2}$$

The linear solution is stable in this case if and only if the real parts of $v > 0, \mu > 0$, and otherwise it is unstable.

3.2 Non-linear Solutions

To determine the stability of the fixed points, one lets

$$a = a_{10} + a_{11} \text{ and } \varphi = \varphi_{10} + \varphi_{11} \tag{24}$$

where a_{10}, φ_{10} are solutions of Eqs. (17), (18) and a_{11}, φ_{11} are perturbations which are assumed to be small comparing to a_{10}, φ_{10}. substituting Eq.(24) into Eqs.(15) and (16) using Eqs. (17) and (18) and keeping only the linear terms in a_{11}, φ_{11} we obtain:

$$a'_{11} = (\frac{-\mu}{2} - \frac{3\beta}{8} a_{10}^2 - \frac{v}{2}) a_{11} + (\frac{f}{4} a_{10} \cos\varphi_{10})\ \varphi_{11} \tag{25}$$

$$\varphi'_{11} = (-\frac{9}{8}(\alpha - \gamma) a_{10} + \frac{f}{4a_{10}} \cos\varphi_{10}) a_{11} + (\frac{f}{4a_{10}} \sin\varphi_{10})\ \ \varphi_{11} \tag{26}$$

$$\begin{vmatrix} \frac{-\mu}{2} - \frac{3\beta}{8} a_{10}^2 - \frac{v}{2} & \frac{f}{4} a_{10} \cos\varphi_{10} \\ -\frac{9}{8}(\alpha - \gamma) a_{10} + \frac{f}{4a_{10}} \cos\varphi_{10} & \frac{f}{4a_{10}} \sin\varphi_{10} \end{vmatrix} = 0 \tag{27}$$

Consequently, a non-linear solution is stable if and only if the real parts of both Eigen values of the coefficient matrix (27) are less than zero.

4. NUMERICAL RESULTS

The dynamical system (1) is solved numerically by using fourth order Runge Kutta method at non resonance case without control as shown in figure 1 from this figure the system is stable with amplitude about 0.15 which considered as basic case. From figure 2 we can see that the system at resonance case $\phi = 2$ without control the amplitude is increased to about 0.85 (six times) compared with the basic case in figure 1. The effect of negative linear feedback velocity at resonance $\phi = 2$ is shown in figure 3 the amplitude is reduced to minimum value. In figure 4 the effect of negative cubic feedback velocity at resonance $\phi = 2$ make the system is stable with amplitude about 0.15, which mean that the negative linear feedback control is more effective than the negative cubic feedback control. Figure 5, shows the effects of different parameters, we can see that the amplitude is monotonic decreasing function of linear damping μ, the damping coefficient, the curvature nonlinearity coefficients α, the inertia nonlinearity coefficients β and γ as shown in Figure 5a-5d respectively. But the amplitude of vibration is monotonic increasing function of the excitation force amplitude f as shown in Figure 5e.

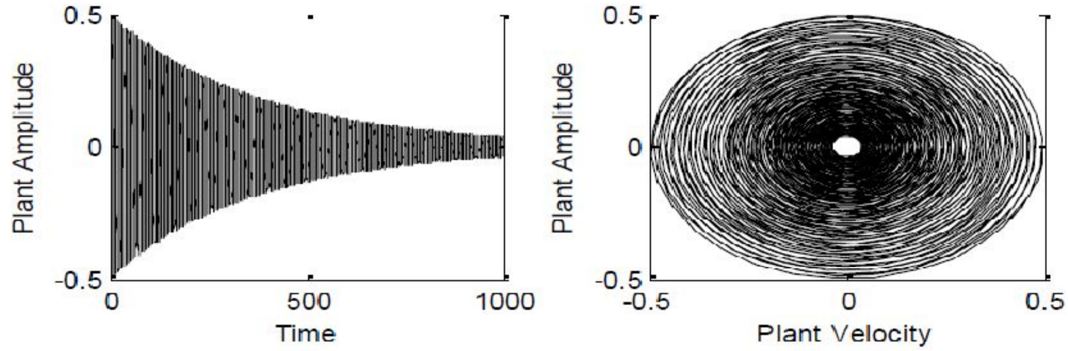

Figure 1: System Behavior without Control at Non Resonance case (Basic Case)

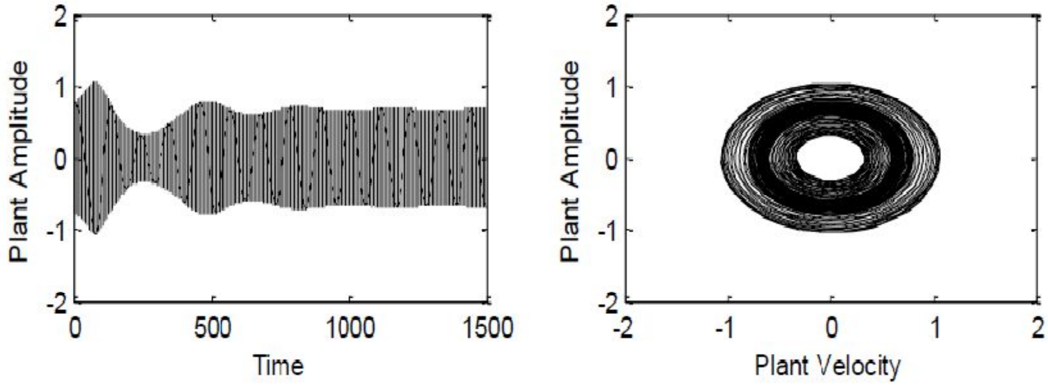

Figure 2: System Behavior without Control at Resonance Case $\phi = 2$

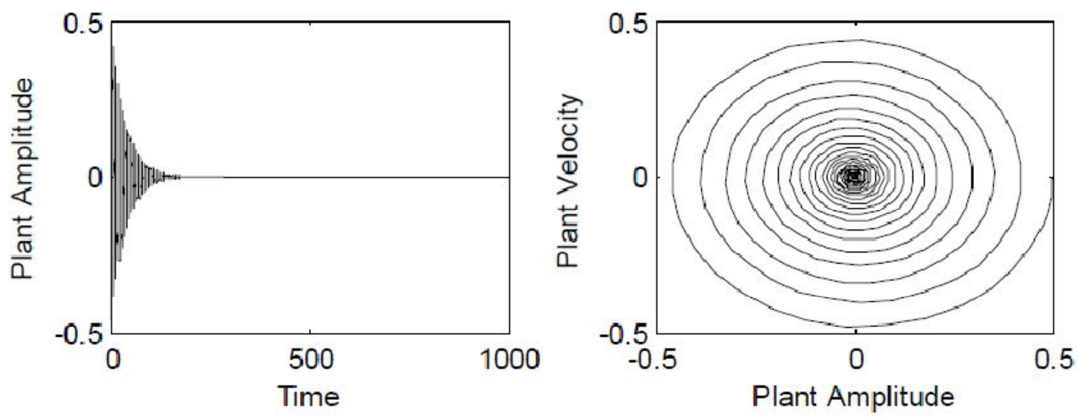

Figure 3: System Behavior with Negative Linear Feedback Velocity at Resonance $\phi = 2$

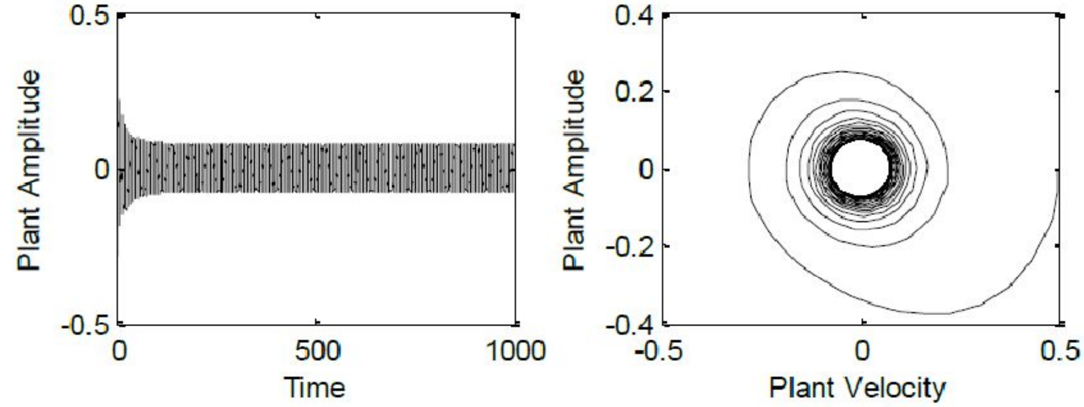

Figure 4: System Behavior with Negative Cubic Feedback Velocity at Resonance $\phi = 2$

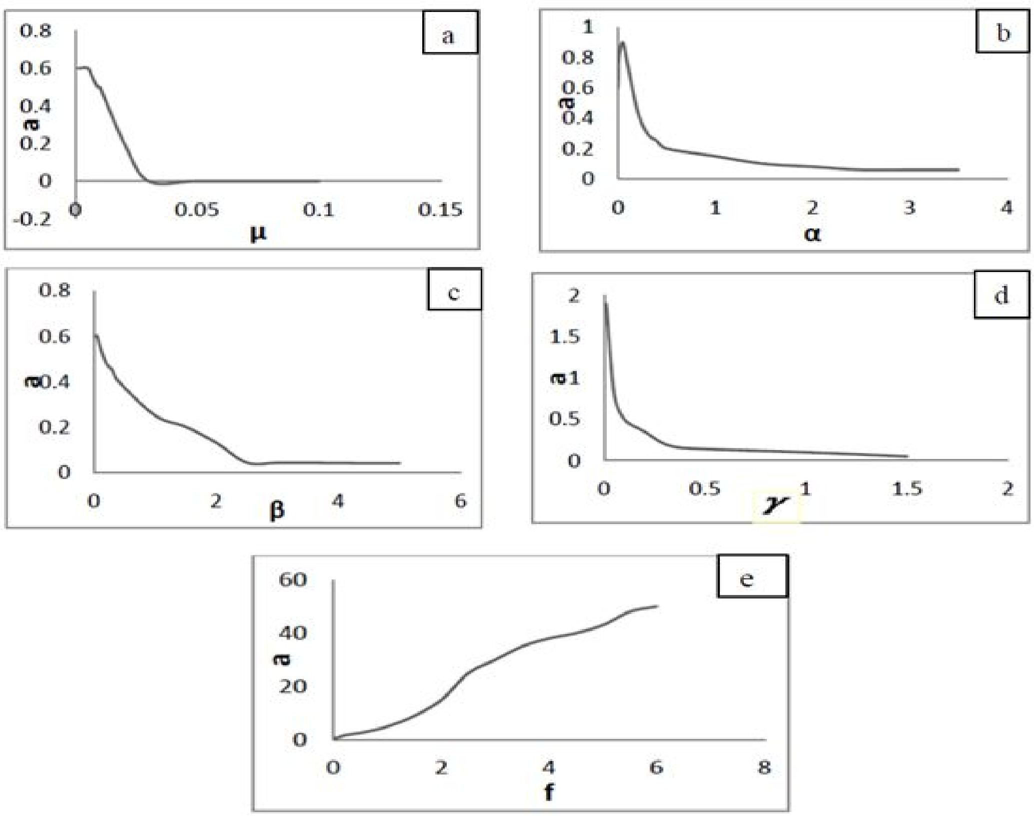

Figure 5: Effect of Parameters without Control

4.1 Frequency Response Curves

The frequency response equation (19) is a non-linear algebraic equation in the amplitude a. This equation is solved numerically and the results are shown in the Figure 6, which representing the vibration of the amplitude a against the detuning parameter σ for the given values of the other parameters. The steady state amplitude is monotonic deceasing function of the gain ν , the linear damping μ, the curvature nonlinearity coefficients α, the inertia nonlinearity coefficients β and γ as shown in Figure 6a, 6d, 6eand 6f respectively. But the state amplitude is monotonic increasing function of the excitation force amplitude f as shown in Figure 6c, which are a good agreement with the numerical solution in Figure 5.

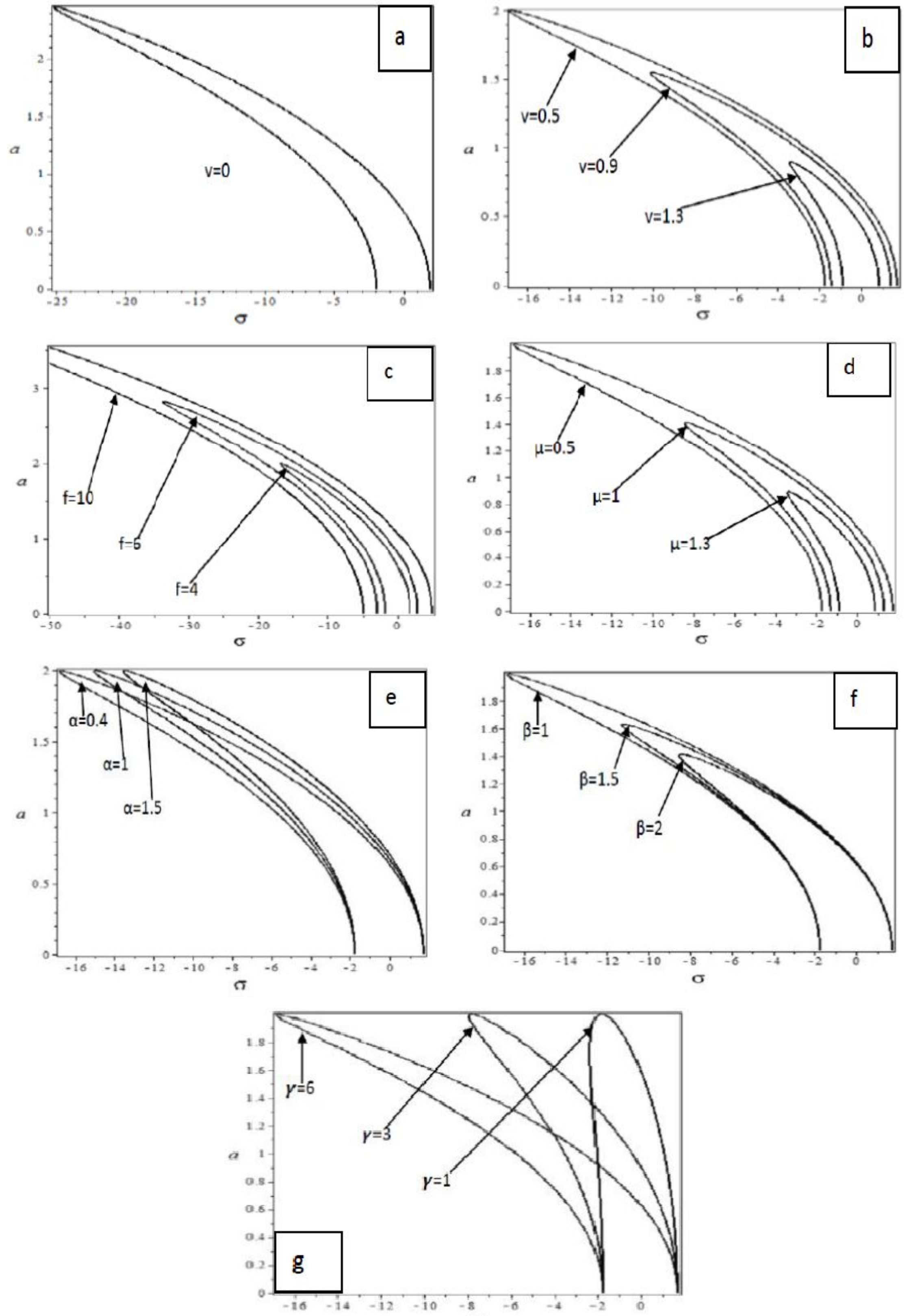

Figure 6: Frequency Response Curve for f=4, α=0.4, β=1, γ=6, μ=0.5, ν=0.5, τ=1

5. COMPARISION BETWEEN NUMERICAL SOLUTION AND APPROXIMATE SOLUTION

Now, we get a good agreement of the approximate solution obtained from frequency response equation and the numerical solution obtained by Runge-Kutta-fourth-order method as shown in Figure 7.

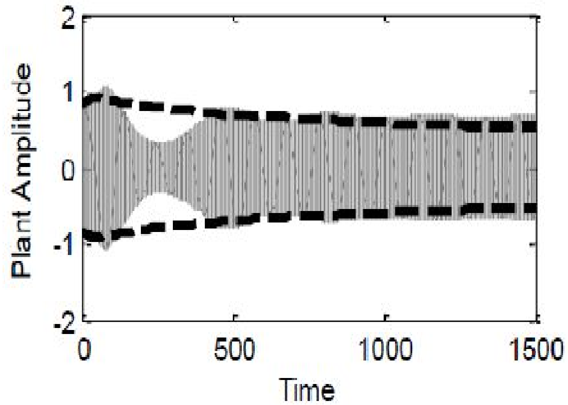

Figure 7: Comparison between the Numerical Solution (______) and the Approximate Solution (--------)

6. CONCLUSIONS

The dynamical behavior of a cantilever beam subject to parametric excitation is investigated. The method of multiple scale perturbation technique is applied to obtain the solution up to the second order approximation. At first we studied the system behavior without control and the effect of parameters at this case. The active control of negative feedback with linear and cubic velocity is used to vibration. control. All resonance cases are studied numerically to obtain the worst case. Stability of the steady state solution for the selected resonance case is studied applying Rung-Kutta fourth method and frequency response equation. From the above study the following may be concluded:

- The worst resonance case is the sub harmonic resonance $\phi \cong 2$, which leads to the amplitudes of the main system are increased to about 0.85 (6 times) of the basic case.
- The linear feedback velocity is better than the cubic feedback velocity to reduce the amplitude of vibration.
- The steady state amplitude is monotonic deceasing function of the gain The steady state amplitude is monotonic deceasing function of the gain ν , the linear damping μ, the curvature nonlinearity coefficients α, the inertia nonlinearity coefficients β and γ. But the state amplitude is monotonic increasing function of the excitation force amplitude f.
- The is a good agreement between the numerical solution and the approximate solution.

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